MATH 6380T, Spring 2021: Topics in Applied Dynamical Systems

Instructor
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Office Hour by appointment.

Meeting Time and Venue
Lectures: Wed & Fri 9am to 10:20am

Venue: (up to Feb 17) Zoom meeting link from Canvas

Course description

Many problems in science and engineering are better understood as a dynamical system with internal mechanisms to evolve in time. This course illustrates how theories from dynamical systems can help analyze and interpret data with some recent developments. Particular emphasis will be given to the scenario where we have limited observations of a system consisting of a large number of variables, such as in the brain.

We will first review classic stability and bifurcation theories and then discuss topics including: theory and application of chaos, data-driven modeling and DMD, and fitting and interpreting RNNs. The students will also have ample opportunity for hands-on practice of the discussed algorithms. The course is suited for both postgraduate and advanced undergraduate students.

Prerequisite
There is no formal prerequisite. However, the students are expected to be familiar with linear algebra, multivariate calculus, ordinary differential equations, basic probability, and statistics, and have experience in some programming language (e.g., python, MATLAB).

Exclusion
None

Outline

- Qualitative analysis of ODEs
  - Fixed points and stability, Phase portrait, Invariant manifolds, Hartman-Grobman theorem, Lyapunov function, Bifurcations, Limit cycle, Poincare map, Poincare-Bendixson theorem, Hopf bifurcation
- Chaos
  - Lorentz system, Strange attractor, Fractal dimension, Lyapunov exponent, Route to chaos, Universality, Takens embedding, Reservoir computing
- Control theory
  - LTI system, State space model, Transfer function, Feedback control, Observability and controllability, Observers, Optimal control, Reduced order models
- Data-driven Methods

Dynamic mode decomposition, Neural networks, BPTT, Recursive least square, FORCE learning, Learning chaotic systems, Examing learned systems

Intended Learning Outcomes

Upon successful completion of this course, students should be able to:
1. Know the fundamental theories and phenomena of dynamical systems
2. Know the main ways and challenges of how dynamical systems are used the in application
3. Gain practical skills to numerically study dynamical systems and interact with data

Evaluation

Credit points: 3

Assessment	Course ILOs
Homework 50%	1,2,3
Journal presentation 15%	1,2
Final exam 35%	1,2,3

- Homework includes derivations, proofs, and numerical (programming) problems
- Each student will present a journal article related to the course topics during the last week of class. You can select the paper yourself. We will also provide some recommendations later.
- Final exam format: 24 hours, open book, take-home exam.

Canvas:
We will use Canvas to distribute and collect the homework and exam. We will also send course announcements via Canvas. So please turn on notifications or check regularly. When you have a question, you are encouraged to post it under Discussion on Canvas, so others can see it and may also help to answer. The instructor will also check and answer the Discussion regularly.

Notes: You can and are encouraged to discuss and collaborate with classmates on homework, as long as each person submits his/her own answers (copy-and-paste is not allowed). For the final exam, however, you are not allowed to discuss it with anyone.

Class participation:
Please turn on your camera and ask questions on your microphone. Please interrupt at any time whenever you find something unclear, chances are others also have the same questions.

References

Lecture slides will be made available after the class.

Optional reference textbook (reserved or available online at HKUST library):

Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering by Strogatz

Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields by Guckenhemier and Holms

Data-Driven Science and Engineering by Brunton and Kutz